Weekly Quiz 1

Student Edition

Divisibility and the Euclidean algorithm

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Use the Euclidean algorithm to compute $\gcd(252, 198)$. **[3 points]**

2. Explain why $\gcd(a, b) = \gcd(b, a - b)$ for positive integers $a > b$. **[3 points]**

3. A 252 cm $\times$ 198 cm rectangle is tiled by the largest possible congruent squares, with no cutting. Find the side length and the number of squares. **[4 points]**

Weekly Quiz 2

Student Edition

Bézout identities and Diophantine equations

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Find integers x, y such that $527x + 341y = \gcd(527, 341)$. **[4 points]**

2. Solve $35x + 22y = 1$ in integers, and give all solutions. **[3 points]**

3. Decide whether $42x + 30y = 18$ has integer solutions. If it does, find one. **[3 points]**

Weekly Quiz 3

Student Edition

Primes and unique factorization

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Factor 756 and 630 into primes; then find their gcd and lcm. **[4 points]**

2. Let p be prime. Use Euclid's lemma to show that $p \mid a^2$ implies $p \mid a$. **[3 points]**

3. The statement "if $p \mid ab$, then $p \mid a$ and $p \mid b$" is false. Give a counterexample and state the correct theorem. **[3 points]**

Weekly Quiz 4

Student Edition

Continued fractions and best approximations

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Evaluate the finite continued fraction $[3; 2, 4]$. **[3 points]**

2. Use Euclid's algorithm to find the simple continued fraction of $43/13$, and list its convergents. **[4 points]**

3. The first four convergents of $\sqrt{2} = [1; 2, 2, \ldots]$ are $1, 3/2, 7/5, 17/12$. Verify the Pell-type identity for the last convergent and explain what it says about the approximation. **[3 points]**

Weekly Quiz 5

Modular arithmetic

Student Edition

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Find the least nonnegative residue of -137 modulo 12. **[2 points]**

2. Solve $7x \equiv 5 \pmod{12}$. **[3 points]**

3. Find the inverse of 17 modulo 43 using the Euclidean algorithm. Check your answer. **[5 points]**

Weekly Quiz 6

Student Edition

The Chinese remainder theorem

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Solve $x \equiv 2 \pmod 3$ and $x \equiv 3 \pmod 5$. Give the complete congruence class. **[3 points]**

2. Solve $x \equiv 1 \pmod 4$ and $x \equiv 3 \pmod 7$. **[4 points]**

3. Explain immediately why $x \equiv 2 \pmod 6$ and $x \equiv 4 \pmod 9$ has no solution. **[3 points]**

Weekly Quiz 7

Student Edition

Necklace coloring and Fermat's little theorem

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Five beads arranged in a circle are colored with three labeled colors. How many nonconstant color words are there, and how many rotation orbits do they form? **[4 points]**

2. Why does every nonconstant word of prime length p have exactly p distinct rotations? **[3 points]**

3. Turn the orbit count into a congruence, then use it to compute $2^{11} \pmod{11}$. **[3 points]**

Weekly Quiz 8

Student Edition

Groups and the standard proof of Fermat's theorem

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. List the units modulo 10 and the multiplicative inverse of each. **[3 points]**

2. List, in order, the elements generated by 3 in the multiplicative group modulo 7. What is the order of 3? **[3 points]**

3. Let $p \nmid a$. Prove $a^{p-1} \equiv 1 \pmod{p}$ by multiplying the nonzero residue classes by a. **[4 points]**

Weekly Quiz 9

Student Edition

Binomial coefficients and divisibility

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. Compute $\binom{8}{3}$ and explain its subset interpretation. **[3 points]**

2. If p is prime and $1 \le k \le p-1$, prove that $p \mid \binom{p}{k}$. **[4 points]**

3. Find the exponent of 2 in $\binom{10}{4}$, either by factoring or by counting binary carries. **[3 points]**

Weekly Quiz 10

Student Edition

Central binomial coefficients and primes

20 minutes • 10 points • Show the essential steps.

Name: _____ **Date:** _____

1. For $n = 4$, verify
$$\frac{4^n}{2n+1} \le \binom{2n}{n} \le 4^n.$$
[3 points]

2. Show that every prime p with $n < p \le 2n$ divides $\binom{2n}{n}$. **[4 points]**

3. Deduce the logarithmic estimate
$$\sum_{n<p\le 2n} \log p \le 2n \log 2,$$
and say in one sentence why this connects binomial coefficients to prime counts. **[3 points]**